

Feed-forward control scheme generate Bell states and three-qubit W-type states when qubits passes through decoherence channel

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It is known that maximally entangled Bell state and three-qubit W-type states are very useful in various quantum information processing task. Thus the problem of preparation of these type of states is very important in quantum information theory. But the factor which prohibit the generation of the above mentioned pure states shared between two and three distant partners is decoherence. When we send one qubit, from a two qubit state, through decoherence channel like amplitude damping channel, the purity of the qubit is lost and it ends up with a mixed state. Therefore it is very difficult to keep the pure maximally entangled state in a maximally entangled pure state or in an entangled state with high entanglement. In this work we have provided a method by which one can generate experimentally a maximally entangled Bell states shared between distant parties with a non-zero probability when a qubit, from a two qubit general state, passes through decoherence channel. Therefore, despite of the fact that qubit is interacting with the noisy channel, we are able to generate Bell state shared between two distant partners. Further, we have shown that it is possible to generate pure three-qubit W-type states shared between three distant partners using economical quantum cloning machine and weak-measurement based feed-forward control scheme, even though the second and third qubit is interacting with the noisy channel. Lastly, we have shown that the generated three-qubit W-type states can be used in teleporting one of the two non-orthogonal states.

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I. INTRODUCTION

Quantum entanglement [1] is such a feature of quantum mechanics which has no classical analogue. The entangled resource state may have bipartite or multipartite entanglement. Quantum information processing tasks which were initially introduced for bipartite system can be extended to multipartite system later. In bipartite system either the state is entangled or separable and if the state is entangled then it is genuine entangled state. Bell states are maximally entangled states and it is very useful in various quantum information processing tasks. Unlike two qubit states, the multipartite entangled states can be classified according to various schemes [2, 3]. Three-qubit states have been classified according to stochastic local operation and classical communication (SLOCC) into six categories. Two of these categories have genuine tripartite entanglement, viz. GHZ-states and W-states [4]. 3-tangle is one of the measures by which one can distinguish GHZ-states and W-states [5]. For GHZ-states, 3-tangle is non-zero while for W-states, it is zero. Previously it was known that W-type states cannot be used for teleportation and superdense coding but Agrawal and Pati [6] introduced a new class of W-type states which are useful in teleportation and superdense coding.

Since entangled states play a major role in various quantum information processing tasks such as quantum teleportation [7], superdense coding [8], remote state preparation [9, 10], secret sharing [11], telecloning [12] and quantum cryptography [13–15] so its generation and manipulation is very important in quantum information theory. There are schemes based

on unitary dynamics for the generation of entangled states [16, 17]. Beside these schemes there are methods of generating entanglement by measurements [18, 19]. It is very difficult to store the generated entangled states by measurement-alone approach. This problem can be resolved by the technique of quantum feedback control [20–23]. An experiment was proposed to generate and stabilize entanglement between two qubits in circuit QED [24, 25]. An experimental demonstration of the generation of superconducting two-qubit Bell state by feedback based on parity measurements is presented in [26]. S-Y Huang et.al. [27] recently presented a simple measurement and feedback control scheme feasible with current circuit QED technology to produce and stabilize the W state $\frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$.

In this work our task is to produce two-qubit Bell states shared between two distant parties and three-qubit W-type states shared between three parties. We assume here that a general two qubit entangled state is prepared in the Alice's laboratory. When Alice want to share one qubit, from the general two qubit state, with Bob, she has to sent the qubit through decoherence channel. In general when qubit is interacted with the noisy environment, it loses its purity and becomes a mixed state. Although one qubit is sent through decoherence channel, we have shown that Alice and Bob manage to share a maximally entangled Bell states by following C-Q Wang et.al. [28] weak-measurement-based feed-forward control scheme. Starting from the generated Bell state shared between two distant partners Alice and Bob, a three-qubit W-type state is prepared by using economical quantum cloning machine and weak-measurement-based feed-forward scheme. The generated three qubit state is not a mixed state but a pure three-qubit W-type state, which is shared between three distant partners Alice, Bob and Charlie. Later we have shown that the generated pure three-qubit W-type state can be used

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in the teleportation of two non-orthogonal states.

The plan of the paper is as follows: In section 2, we revisit C-Q Wang et.al. weak-measurement-based feed-forward control scheme. In this scheme, authors showed that how one can protect the purity of the qubit when passing through decoherence channel. In section 3, we have used the concept discussed in section 2 to generate two-qubit Bell states shared between sender and the receiver located far away from each other, when a qubit passes through amplitude damping channel. In section 4, we have shown that a particular form of three-qubit W-type state is produced by economical quantum cloning machine and weak-measurement-based feed-forward control scheme. In section 5, the generated three-qubit W-type state is used to teleport two non-orthogonal states. Finally, we conclude in section 6.

II. C-Q WANG ET.AL. WEAK-MEASUREMENT-BASED FEED-FORWARD CONTROL SCHEME

C-Q Wang et.al. [28] introduced a feed-forward control scheme to protect an unknown quantum state. The scheme is based on one complete pre-weak measurement, two incomplete post-weak measurements and two feed-forward operations and their reversals. The scheme of protecting unknown quantum states when it passes through decoherence channel, goes as follows: Let us consider an unknown quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1 \quad (1)$$

Firstly, we perform a pre-weak measurement on the state $|\psi\rangle$ in (1) before it passes through the noisy channel. Pre-weak measurement can be chosen as $\Pi_1 = M_1^\dagger M_1$ and $\Pi_2 = M_2^\dagger M_2$, and M_1 and M_2 are given by

$$M_1 = \begin{pmatrix} \sqrt{p} & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, M_2 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{p} \end{pmatrix} \quad (2)$$

where p is the pre-weak measurement strength and $\sum_{i=1}^2 M_i^\dagger M_i = I$.

If the measurement outcome is M_1 then we adopt the feed-forward operation $F_1 = I$ but if the measurement outcome is M_2 , we adopt the feed-forward operation $F_2 = \sigma_x$. In this work we explain the whole protocol when the measurement outcome is M_1 . In a similar fashion, we could demonstrate the whole protocol with measurement outcome M_2 also.

The occurrence of the measurement outcome M_1 reduces the state $|\psi\rangle$ to

$$|\psi\rangle_{M_1} = \frac{M_1|\psi\rangle}{\sqrt{\langle\psi|\Pi_1|\psi\rangle}} = \frac{1}{N_{M_1}}(\alpha\sqrt{p}|0\rangle + \beta\sqrt{1-p}|1\rangle) \quad (3)$$

where $N_{M_1} = \langle\psi|\Pi_1|\psi\rangle = |\alpha|^2 p + |\beta|^2 (1-p)$. We perform the feed-forward operation $F_1 = I$ on $|\psi\rangle_{M_1}$ which keeps the state as it is. Qubit (3) is then passed through the noisy channel for a period of τ . The qubit is then no longer pure because of energy relaxation with the rate Γ . To keep the qubit in a pure state, we disentangle the relaxation into "jump" and "no jump" scenarios. When the qubit is passing through the

amplitude damping channel, the qubit trajectories is divided into two parts: (i) the qubit is jumping into the state $|0\rangle$ with the "jump" probability $P^j = N_{M_1}|\beta|^2(1 - e^{-\Gamma\tau})$. (ii) "no jumping" state of the qubit is

$$|\psi\rangle_{nj} = \frac{1}{\sqrt{P^{nj}}}(\alpha\sqrt{p}|0\rangle + \beta\sqrt{1-p}e^{-\frac{\Gamma\tau}{2}}|1\rangle) \quad (4)$$

where $P^{nj} = |\alpha|^2 p + |\beta|^2 (1-p)e^{-\Gamma\tau}$. Then the reversed feed-forward operation $F_1 = I$ retain the state $|0\rangle$ and $|\psi\rangle_{nj}$. Lastly, we measure the qubit by post-weak measurement $\bigwedge = O_1^\dagger O_1$, where

$$O_1 = \begin{pmatrix} \sqrt{1-p_1} & 0 \\ 0 & 1 \end{pmatrix}, \quad (5)$$

p_1 is the post-weak measurement strength.

The measured state from the "jumping" trajectory is $|0\rangle$ with probability $P^{jN_1} = N_{M_1}|\beta|^2(1 - e^{-\Gamma\tau})(1 - p_1)$. The measured state from the "no jumping" trajectory is given by

$$|\psi\rangle_{njn1} = \frac{1}{\sqrt{P^{njN_1}}}(\alpha\sqrt{p}\sqrt{1-p_1}|0\rangle + \beta\sqrt{1-p}e^{-\frac{\Gamma\tau}{2}}|1\rangle) \quad (6)$$

where $P^{njN_1} = |\alpha|^2 p(1 - p_1) + |\beta|^2 (1-p)e^{-\Gamma\tau}$. If we choose the post-weak measurement strength as

$$p_1 = 1 - \frac{(1-p)e^{-\Gamma\tau}}{p} \quad (7)$$

then the state in the "no jumping" trajectory will be the same as initial state. The success probability of retaining the initial state is given by

$$P^S = (1-p)e^{-\Gamma\tau} \quad (8)$$

III. GENERATION OF TWO-QUBIT BELL STATES WHEN A QUBIT PASSES THROUGH AMPLITUDE DAMPING CHANNEL

Let us start with two qubit entangled state

$$|\Psi_g^{AA}\rangle = |0\rangle_1 \otimes (\alpha|0\rangle_2 + \beta|1\rangle_2) + |1\rangle_1 \otimes (\gamma|0\rangle_2 + \delta|1\rangle_2), \\ |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1, \alpha \neq \gamma, \beta \neq \delta \quad (9)$$

If $\alpha = \gamma$ and $\beta = \delta$ then the state (9) would become a product state so we have taken $\alpha \neq \gamma, \beta \neq \delta$.

Initially both qubit possessed by Alice. Alice then perform a pre-weak measurement $I \otimes M_1$ on the second qubit followed by feed-forward operation $I \otimes F_1$. Alice then send the second qubit to Bob through amplitude damping channel. The amplitude damping channel can be described by the Kraus operators as

$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{pmatrix}, K_2 = \begin{pmatrix} 0 & \sqrt{r} \\ 0 & 0 \end{pmatrix} \quad (10)$$

where r is the magnitude of the decoherence. Taking the argument from the previous section, the qubit trajectory can be

divided into "jump" and "no jump" trajectory. When the second qubit is in "jumping" trajectory and Bob operate reversed feed-forward operation $I \otimes F_1^{-1}$ and a partial weak measurement $I \otimes O_1$ on his qubit, the shared state between Alice and Bob is given by

$$|\Psi_j^{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{12} + |10\rangle_{12}) \quad (11)$$

Clearly the state (11) is a product state and it is not useful in any quantum information processing task.

When the second qubit is in "no jumping" trajectory and Bob operate reversed feed-forward operation $I \otimes F_1^{-1}$ and a partial weak measurement $I \otimes O_1$ on his qubit, the shared state between Alice and Bob takes the form

$$|\Psi_{nj}^{AB}\rangle = a|00\rangle_{12} + b|01\rangle_{12} + c|10\rangle_{12} + d|11\rangle_{12}, \quad (12)$$

where the qubit 1 is with Alice and qubit 2 is with Bob. The parameters a, b, c, d is given by

$$\begin{aligned} a &= \frac{\alpha\sqrt{p}\sqrt{1-p_1}}{\sqrt{2P^{nj}N_1}}, b = \frac{\beta\sqrt{1-pe^{-\frac{\Gamma\tau}{2}}}}{\sqrt{2P^{nj}N_1}} \\ c &= \frac{\gamma\sqrt{p}\sqrt{1-p_1}}{\sqrt{2P^{nj}N_1}}, d = \frac{\delta\sqrt{1-pe^{-\frac{\Gamma\tau}{2}}}}{\sqrt{2P^{nj}N_1}} \end{aligned} \quad (13)$$

where $P^{nj}N_1 = |\gamma|^2 p(1-p_1) + |\delta|^2 (1-p)e^{-\Gamma\tau}$, $P^{nj}N_1 = |\alpha|^2 p(1-p_1) + |\beta|^2 (1-p)e^{-\Gamma\tau}$.

If we assume $1 - e^{-\Gamma\tau} = r$ then the decoherence channel is the same as amplitude damping channel.

The Hadamard transformation is given by

$$\begin{aligned} |0\rangle &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ |1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned} \quad (14)$$

Bob perform Hadamard transformation on his qubit. As a result of the transformation, the two-qubit state (12) reduces to

$$|\Psi_{njh}^{AB}\rangle = \frac{1}{\sqrt{2}}((a-b)|00\rangle_{12} + (a+b)|01\rangle_{12} + (c-d)|10\rangle_{12} + (c+d)|11\rangle_{12}), \quad (15)$$

If we choose $\beta = -\alpha$, $\delta = \gamma$ and the post-weak measurement strength p_1 as given in (7) then the state $|\Psi_{njh}^{AB}\rangle$ becomes maximally entangled Bell state i.e. $\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})$. The probability of generating the maximally entangled Bell state with this procedure is $\frac{1}{2}$.

IV. GENERATION OF THREE-QUBIT W-TYPE STATE WHEN SECOND AND THIRD QUBIT PASSES THROUGH AMPLITUDE DAMPING CHANNEL

In the previous section, we have seen how two-qubit maximally entangled Bell state is generated when the second qubit is passing through the amplitude damping channel. In this

section, we will start with this maximally entangled Bell state shared between Alice and Bob

$$|\psi\rangle_{Bell}^{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \quad (16)$$

Now our task is to generate three-qubit state from two-qubit Bell state (16). To do this, we are using economical quantum cloning machine. If cloning machine does not need any ancilla then it is called economical quantum cloning machine. Economical quantum cloning transformation is given by

$$\begin{aligned} U|0\rangle|0\rangle &= |0\rangle|0\rangle \\ U|1\rangle|0\rangle &= \cos\alpha|1\rangle|0\rangle + \sin\alpha|0\rangle|1\rangle \end{aligned} \quad (17)$$

Bob then apply economical quantum cloning transformation to his qubit. As a result of the cloning transformation, a three-qubit state is generated and is given by

$$\begin{aligned} |\psi\rangle_{ABC} &= \frac{1}{\sqrt{2}}(|000\rangle_{ABC} + \cos\alpha|110\rangle_{ABC} \\ &\quad + \sin\alpha|101\rangle_{ABC}) \end{aligned} \quad (18)$$

where the qubit A hold by Alice while qubit B and C possessed by Bob.

Non-maximally Hadamard transformation is given by

$$\begin{aligned} |0\rangle &\rightarrow u|0\rangle + v|1\rangle \\ |1\rangle &\rightarrow v|0\rangle - u|1\rangle, \quad u^2 + v^2 = 1 \end{aligned} \quad (19)$$

Equation (19) reduces to hadamard transformation when $u = v = \frac{1}{\sqrt{2}}$.

Bob apply non-maximally Hadamard transformation on qubit C and hence the state $|\psi\rangle_{ABC}$ reduces to

$$\begin{aligned} |\psi\rangle_{ABC}^{nmH} &= \frac{1}{\sqrt{2}}((|00\rangle_{AB} + \cos\alpha|11\rangle_{AB}) \otimes (u|0\rangle_C + v|1\rangle_C) \\ &\quad + \sin\alpha|10\rangle_{AB} \otimes (v|0\rangle_C - u|1\rangle_C)) \end{aligned} \quad (20)$$

Bob now executing feed-forward scheme described in section-II to send the third qubit C to Charlie. When the third qubit C passing through the amplitude damping channel and reached to Charlie, the three-qubit state shared between Alice, Bob and Charlie in "no jumping" trajectory becomes

$$\begin{aligned} |\psi\rangle_{ABC}^D &= u_1|000\rangle_{ABC} + u_2|001\rangle_{ABC} + u_3|110\rangle_{ABC} \\ &\quad + u_4|111\rangle_{ABC} + u_5|100\rangle_{ABC} \\ &\quad + u_6|101\rangle_{ABC} \end{aligned} \quad (21)$$

where the qubit C possessed by Charlie and the coefficients are given by

$$\begin{aligned} u_1 &= \frac{u\sqrt{p}\sqrt{1-p_1}}{\sqrt{2k_1}}, u_2 = \frac{v\sqrt{1-pe^{-\frac{\Gamma\tau}{2}}}}{\sqrt{2k_1}}, \\ u_3 &= \frac{ucos\alpha\sqrt{p}\sqrt{1-p_1}}{\sqrt{2k_1}}, u_4 = \frac{vcos\alpha\sqrt{1-pe^{-\frac{\Gamma\tau}{2}}}}{\sqrt{2k_1}}, \\ u_5 &= \frac{vsin\alpha\sqrt{p}\sqrt{1-p_1}}{\sqrt{2k_2}}, u_6 = \frac{-usin\alpha\sqrt{1-pe^{-\frac{\Gamma\tau}{2}}}}{\sqrt{2k_2}} \\ k_1 &= u^2p(1-p_1) + v^2(1-p)e^{-\Gamma\tau}, \\ k_2 &= v^2p(1-p_1) + u^2(1-p)e^{-\Gamma\tau} \end{aligned} \quad (22)$$

3-tangle of the state $|\psi\rangle_{ABC}^D$ is zero and hence it represent a W - type state which is shared between Alice, Bob and Charlie.

Charlie apply Hadamard transformation on his qubit and the resultant three-qubit state takes the form as

$$\begin{aligned} |\psi\rangle_{ABC}^{DH} = & \frac{1}{\sqrt{2}}[(u_1 + u_2)|000\rangle_{ABC} + (u_1 - u_2)|001\rangle_{ABC} \\ & + (u_3 + u_4)|110\rangle_{ABC} + (u_3 - u_4)|111\rangle_{ABC} \\ & + (u_5 + u_6)|100\rangle_{ABC} \\ & + (u_5 - u_6)|101\rangle_{ABC}] \end{aligned} \quad (23)$$

If we choose the post-weak measurement strength as

$$p_1 = 1 - \frac{v^2(1-p)e^{-\Gamma\tau}}{u^2p} \quad (24)$$

then $u_1 = u_2 = \frac{1}{2}$, $u_3 = u_4 = \frac{\cos\alpha}{2}$, $u_5 = \frac{v^2\sin\alpha}{\sqrt{2(u^4+v^4)}}$, $u_6 = \frac{-u^2\sin\alpha}{\sqrt{2(u^4+v^4)}}$. This reduces the state $|\psi\rangle_{ABC}^{DH}$ to

$$\begin{aligned} |\psi\rangle_{ABC}^W = & \frac{1}{\sqrt{N}}[\frac{1}{\sqrt{2}}|000\rangle_{ABC} + \frac{1}{\sqrt{2}}\cos\alpha|110\rangle_{ABC} \\ & + \frac{(v^2 - u^2)\sin\alpha}{2\sqrt{u^4 + v^4}}|100\rangle_{ABC} \\ & + \frac{\sin\alpha}{2\sqrt{(u^4 + v^4)}}|101\rangle_{ABC}] \end{aligned} \quad (25)$$

where $N = \frac{1}{2} + \frac{\cos^2\alpha}{2} + \frac{((v^2 - u^2)^2 + 1)\sin^2\alpha}{4(u^4 + v^4)}$.

For $u = v = \frac{1}{\sqrt{2}}$, the three-qubit state (25) reduces to

$$\begin{aligned} |\psi\rangle_{ABC}^{W1} = & \frac{1}{\sqrt{2}}[|000\rangle_{ABC} + \cos\alpha|110\rangle_{ABC} \\ & + \sin\alpha|101\rangle_{ABC}] \end{aligned} \quad (26)$$

When Alice apply the Pauli operator σ_x on her qubit then the state $|\psi\rangle_{ABC}^{W1}$ reduces to the state introduced by Agrawal and Pati [6]

$$\begin{aligned} |\psi\rangle_{ABC}^{W2} = & (\sigma_x \otimes I \otimes I)|\psi\rangle_{ABC}^{W1} \\ = & \frac{1}{\sqrt{2}}[|100\rangle_{ABC} + \cos\alpha|010\rangle_{ABC} \\ & + \sin\alpha|001\rangle_{ABC}] \end{aligned} \quad (27)$$

This class of W - type states can be used for perfect teleportation and superdense coding.

V. PERFECT TELEPORTATION OF TWO NON-ORTHOGONAL STATES WITH $|\psi\rangle_{ABC}^{W2}$

In this section we have shown that how two non-orthogonal states can be teleported via a three-qubit state $|\psi\rangle_{ABC}^{W2}$. Let us consider two nonorthogonal states to be teleported is given by

$$|\chi_1\rangle_A = x|0\rangle_A + y|1\rangle_A, \quad x^2 + y^2 = 1 \quad (28)$$

$$\begin{aligned} |\chi_2\rangle_A = & (sx + y\sqrt{1-s^2})|0\rangle_A \\ & + (sy - x\sqrt{1-s^2})|1\rangle_A \end{aligned} \quad (29)$$

We note that $\langle\chi_1|\chi_2\rangle = s$, $0 \leq s \leq 1$. Let us assume that the two single qubit non-orthogonal states defined above are with Alice and she has information about the parameters x and s . She want to teleport the messages encoded in the non-orthogonal states to Bob with the help of three-qubit W - type state

$$\begin{aligned} |\psi\rangle_{AAB}^{W2} = & \frac{1}{\sqrt{2}}[|100\rangle_{AAB} + \cos\alpha|010\rangle_{AAB} \\ & + \sin\alpha|001\rangle_{AAB}] \end{aligned} \quad (30)$$

where the first two qubits are with Alice and the third qubit is with Bob. α is the economical cloning machine parameter. The composite five qubit state is given by

$$|\psi\rangle_{AAAB} = |\chi_1\rangle_A \otimes |\chi_2\rangle_A \otimes |\psi\rangle_{AAB}^{W2} \quad (31)$$

Alice then perform two Bell state measurements on her qubits. As a result of the measurement, either a bit or a qubit is generated at Bob's site. Appearance of bit at Bob's site means no non-orthogonal states appeared at his place and hence this case is considered as failure of the protocol. But the case, when qubit is generated, can be considered as success of the protocol because in this case the generated qubit can be converted into one of the two non-orthogonal states $|\chi_1\rangle_B$ or $|\chi_2\rangle_B$.

Let us consider the following cases:

Case-Ia: If the measurement outcome is $|\psi^+\rangle_{AA} \otimes |\phi^+\rangle_{AA}$ or $|\psi^+\rangle_{AA} \otimes |\phi^-\rangle_{AA}$ then Alice sent two classical bits $|0\rangle \otimes |0\rangle$ to Bob. Bob applies I on the received qubit after getting two classical bits from Alice. Before Bell state measurement, if Alice chooses the cloning machine parameter α in such a way that $\sin\alpha = \frac{K^2-1}{K^2+1}$ and $\cos\alpha = \frac{-2K}{K^2+1}$, where $K = \frac{x(sx+y\sqrt{1-s^2})}{y(sy-x\sqrt{1-s^2})}$ then the qubit appear at Bob's place is $|\chi_1\rangle_B$.

Case-Ib: If the measurement outcome is $|\psi^+\rangle_{AA} \otimes |\phi^+\rangle_{AA}$ or $|\psi^+\rangle_{AA} \otimes |\phi^-\rangle_{AA}$ then Alice sent two classical bits $|0\rangle \otimes |1\rangle$ to Bob. After getting classical bits, Bob applies σ_x on the received qubit. If Alice chooses the cloning machine parameter α in such a way that $\sin\alpha = \frac{\sqrt{2-K^2+K}}{2}$ and $\cos\alpha = \frac{\sqrt{2-K^2-K}}{2}$ and then the qubit appear at Bob's place is $|\chi_2\rangle_B$.

Case-IIa: If the measurement outcome is $|\psi^-\rangle_{AA} \otimes |\phi^+\rangle_{AA}$ or $|\psi^-\rangle_{AA} \otimes |\phi^-\rangle_{AA}$ then Alice sent two classical bits $|1\rangle \otimes |0\rangle$ to Bob. Bob then applies σ_z on the received qubit. If Alice chooses $\sin\alpha = \frac{K^2-1}{K^2+1}$ and $\cos\alpha = \frac{2K}{K^2+1}$ before Bell state measurement then in this case also the state appears at Bob's site is $|\chi_1\rangle_B$.

Case-IIb: If the measurement outcome is $|\psi^-\rangle_{AA} \otimes |\phi^+\rangle_{AA}$ or $|\psi^-\rangle_{AA} \otimes |\phi^-\rangle_{AA}$ then Alice sent a classical bit $|1\rangle \otimes |1\rangle$ to Bob. After getting classical bits, Bob applies $-i\sigma_y$ on the received qubit. If Alice chooses $\sin\alpha = \frac{K+\sqrt{2-K^2}}{2}$ and $\cos\alpha = \frac{K-\sqrt{2-K^2}}{2}$ before Bell state measurement then the state appears at Bob's site is $|\chi_2\rangle_B$.

Case-IIIa: If the measurement outcome is $|\psi^+\rangle_{AA} \otimes |\psi^+\rangle_{AA}$

or $|\psi^+\rangle_{AA} \otimes |\psi^-\rangle_{AA}$ then Alice sent two classical bits $|0\rangle \otimes |0\rangle$ to Bob. Bob applies I on the received qubit After getting two classical bits from Alice. If Alice Chooses $\sin\alpha = \frac{\sqrt{2-L^2}+L}{2}$ and $\cos\alpha = \frac{\sqrt{2-L^2}-L}{2}$, where $L = \frac{x(sy-x\sqrt{1-s^2})}{y(sx+y\sqrt{1-s^2})}$ before Bell state measurement then the state appears at Bob's site is $|\chi_2\rangle_B$.

Case-IIIb: If the measurement outcome is $|\psi^+\rangle_{AA} \otimes |\psi^+\rangle_{AA}$ or $|\psi^+\rangle_{AA} \otimes |\psi^-\rangle_{AA}$ then Alice sent two classical bits $|0\rangle \otimes |0\rangle$ to Bob. Bob applies I on the received qubit After getting two classical bits from Alice. If Alice Chooses in this case $\sin\alpha = \frac{L^2-1}{L^2+1}$ and $\cos\alpha = \frac{-2L}{L^2+1}$ before Bell state measurement then the state appears at Bob's site is $|\chi_1\rangle_B$.

Case-IVa: If the measurement outcome is $|\psi^-\rangle_{AA} \otimes |\psi^+\rangle_{AA}$ or $|\psi^-\rangle_{AA} \otimes |\psi^-\rangle_{AA}$ then Alice sent two classical bits $|1\rangle \otimes |0\rangle$ to Bob. Bob applies σ_z on the received qubit after getting two classical bits from Alice. If Alice Chooses $\sin\alpha = \frac{L+\sqrt{2-L^2}}{2}$ and $\cos\alpha = \frac{L-\sqrt{2-L^2}}{2}$ before Bell state measurement then the state appears at Bob's site is $|\chi_2\rangle_B$.

Case-IVb: If the measurement outcome is $|\psi^-\rangle_{AA} \otimes |\psi^+\rangle_{AA}$ or $|\psi^-\rangle_{AA} \otimes |\psi^-\rangle_{AA}$ then Alice sent two classical bits $|1\rangle \otimes |0\rangle$ to Bob. Bob applies σ_z on the received qubit after getting two classical bits from Alice. If Alice Chooses $\sin\alpha = \frac{L^2-1}{L^2+1}$ and $\cos\alpha = \frac{2L}{L^2+1}$ before Bell state measurement then the

state appears at Bob's site is $|\chi_1\rangle_B$.

VI. CONCLUSION

To summarize, we have made a study of the generation of a pure two-qubit Bell states $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ shared between two distant partners, even though a qubit from a two-qubit general state is interacting with the noisy environment. This would be possible only if we use C-Q Wang et.al. weak-measurement-based feed-forward control scheme. A three-qubit W-type states $\frac{1}{\sqrt{2}}[|100\rangle + \cos\alpha|010\rangle + \sin\alpha|001\rangle]$, where α is the economical quantum cloning machine parameter, can also be prepared by economical quantum cloning machine and feed-forward scheme. We have shown that the generated three-qubit state shared between Alice, Bob and Charlie reside in three different laboratories. Since the two -qubit Bell states and a particular form of three-qubit W-type states discussed in this work are very useful in secret sharing and quantum cryptography so the study of their production is very important in quantum information theory. Further we have shown that one of the two non-orthogonal states can be teleported via W-type states.

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